JOURNAL OF COMPUTATIONAL PHYSICS 87, 73–90 (1990)

A View-Factor Method for Solving Time-Dependent Radiation Transport Problems Involving Fixed Surfaces with Intervening, Participating Media*

DOUGLAS J. DRAKE

KMS Fusion, Inc., P.O. Box 1567, Ann Arbor, Michigan 48106

Received February 8, 1988; revised March 28, 1989

A description is given of a view factor technique for solving time-dependent, non-linear radiative transfer problems. It is believed unique among view factor methods in having the ability to solve problems involving gases or plasmas with emissive and absorptive characteristics which vary in space and time. The radiative characteristics of the enclosing surfaces can also have an arbitrary space and time dependence, and these characteristics can be updated based on an appropriate physical model of the interaction of the background radiation field with the medium. Various sample problems are solved, including a time-dependent solution for concentric spheres filled with a participating medium of arbitrary opacity for a step function source. © 1990 Academic Press, Inc

1. INTRODUCTION

View factor techniques, which have typically been applied to steady-state radiative heat transfer problems in the past, have recently been used successfully [1, 2, 3], to solve time-dependent radiative transfer problems. Such methods, in general, involve the subdivision of given surfaces into small surface elements, either manually or with the aid of a computer algorithm, the specification of the radiative characteristics of the surface elements so generated, and the modeling of the radiative transfer between the surfaces with the aid of view factors. They can be applied where complex radiation source dependence on time and space make application of competing methods, such as Monte Carlo, extremely costly in terms of computer time and memory. Alternatively, they can be used to provide quick iterative solutions to steady-state or near steady-state problems in situations involving complicated geometries and source functions.

Unfortunately, to date the use of view factor techniques and computer codes has been limited mainly to situations in which there is no participating medium, such as a gas or plasma. For vacuum problems, the fraction of radiation emitted by one diffusely emitting surface which eventually strikes another surface is merely a function of the source strength at the emitting surface and the areas and orientations

* Prepared for the Department of Energy under Contract DE-AC08-87DP10560.

0021-9991/90 $3.00

of the two surfaces. If some intervening medium is included, however, it is necessary to know not only the emissive and absorptive characteristics of the medium and how it varies over some large number of time steps, but how these characteristics are affected by the radiation field. Furthermore, some technique must be developed for determining the contribution of the medium to beams of radiation as they pass from one surface to another. If the realistic assumption is made that the medium is non-homogeneous, and its characteristics can vary more or less drastically throughout the problem space, the complexity of the problem is evident.

Here we describe a view factor method which includes the effects of participating media in time-dependent radiation transport calculations, and the results obtained in applying the method to various sample problems with the aid of the RAYNA II view factor code.

2. View Factor Fundamentals

Given two differential surfaces dA_1 and dA_2 separated by a distance r, the rate at which radiation leaving dA_1 strikes dA_2 is [4]

$$dq_{1-2} = I_1 \cos\theta_1 \, dA_1 \, d\omega_{1-2}, \tag{1}$$

where I_1 is the intensity of radiation from dA_1, θ_1 is the angle formed by the normal to dA_1 and the line joining the two surfaces, $dA_1 \cos\theta_1$ is the projection of area element dA_1 as seen from dA_2, and $d\omega_{1-2}$ is the solid angle subtended by dA_2 as seen from dA_1. Since emissive power E is related to intensity I by

$$E = \pi I \tag{2}$$

and since

$$d\omega_{1-2} = \cos\theta_2 \frac{dA_2}{r^2}, \tag{3}$$

where θ_2 is defined similarly to θ_1 and r is the distance between the two surfaces, one has

$$dq_{1-2} = E_1 \, dA_1 \left(\frac{\cos\theta_1 \cos\theta_2 \, dA_2}{\pi r^2} \right). \tag{4}$$

The term in parenthesis in Eq. (4) is the fraction of radiation from surface dA_1 that strikes surface dA_2. Then, for two macroscopic geometrical surfaces A_1 and A_2, one can write

$$A_1 F_{1-2} = \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} \, dA_1 \, dA_2, \tag{5}$$

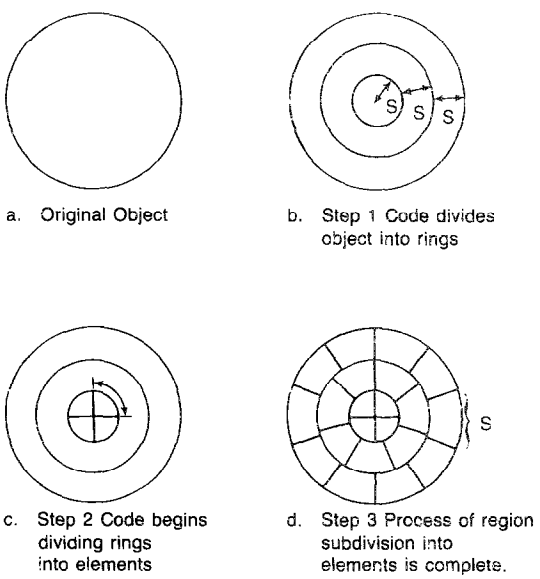
where F_{1-2} is called the view factor evaluated on the basis of area A_1.

Typically, users of view factor computer codes approximate the differential surface elements described above by subdividing the surfaces which make up a problem into some chosen number of elements of approximately equal area, either manually or automatically with the aid of a computer algorithm. The actual number of elements employed can normally be specified by the user, limited by machine speed and memory constraints. The set of geometric surfaces presently available to RAYNA II users includes disks, cylinders, cones, spheres, and various derivatives thereof. The algorithm for subdividing these surfaces into smaller elements is similar in all cases. First, the surface is divided into rings, which are further subdivided into elements, as shown below for the case of a disk (Fig. 1).

Obviously, the rings into which the disk has been subdivided by the code, as shown in Fig. 1b, all have a common axis of symmetry. In fact, the code will generate such axisymmetric rings in the process of breaking down all of the other surfaces in its repertoire, a circumstance which will prove useful in exploiting 2D symmetry when it exists.

At this point surface-to-surface view factors between the elements must be calculated. Readers interested in the details of such calculations may consult Refs. [4 or 5], which contain a review of view factor techniques. A check must be made to determine if the optical path between any pair of elements is obstructed by an intervening surface, in which case the view factor between the elements is reset to zero.

The time dependence of the problem is modeled by causing the calculation to proceed in discrete time steps. Once element to element view factors have been



a. Original Object

b. Step 1 Code divides object into rings

c. Step 2 Code begins dividing rings into elements

d. Step 3 Process of region subdivision into elements is complete.

FIG. 1 RAYNA II Algorithm for region subdivision as applied to a disk.

computed, solution of the radiative transfer problem consists of determining the contribution of each of the elements to every othe element in the problem during each time step. If it takes light n time steps to travel from element j to element k, the radiation contributed by j to k in a time step will be

$$E_{jk} = S_j[(N-n)\,\Delta t]\,A_j\,F_{jk}\,\Delta t, \tag{6}$$

where E_{jk} is the radiative energy contribution of element j to element k in the time step in ergs, N is the number of time steps since the beginning of the problem, $S_j[(N-n)\,\Delta t]$ is the radiative source strength of element j during time step $(N-n)$ in ergs/cm^2-s, A_j is the area of j, F_{jk} is the view factor from j to k, and Δt is the time step size. Radiative source strength, S_j, is recomputed every time step based on the total amount of incoming radiation and user-declared boundary conditions, including surface albedo or reflectivity and legislated or computed surface source strengths. Evidently, one can solve the vacuum problem by retaining in memory the radiative source strengths for some number of time steps equal to the time it takes for light to travel between the two most distant elements in the problem, the element areas and the view factors between each pair of elements which are visible to each other. In essence, one keeps track of the "bundles" of radiation mentioned earlier as they pass along rays from element to element. This strategy has been used successfully to provide time-dependent solutions for vacuum problems [1, 2].

3. The Transport Problem with Participating Media

If participating media are included, the simple treatment of radiation transport described above is no longer adequate. One must not only determine what fraction of radiation from one surface element is emitted in the direction of another surface element, but what happens to the radiation as it passes through the intervening material. As a minimum, one must set up some sort of volume mesh and specify the emissive and absorptive characteristics of the material at each mesh point. If a multi-group treatment is necessary, this must be done for each radiation energy group. A volume element of known volume must be associated with each mesh point, and an algorithm must be provided to update the physical characteristics of the medium in each element as it emits and absorbs radiation. Finally, one must calculate what volume elements a beam passes through which was emitted by one surface element in the direction of another surface element, and the proportion of the total path length between the two surface elements which is traveled in each of these volume elements.

Generation of a Volume Mesh

Generation of a volume mesh is accomplished with the RAYNA II code as depicted in Fig. 2. First, the code finds a 2D outline or cross section of the problem space. A 2D mesh is set up on this outline by stepping along the axis of symmetry

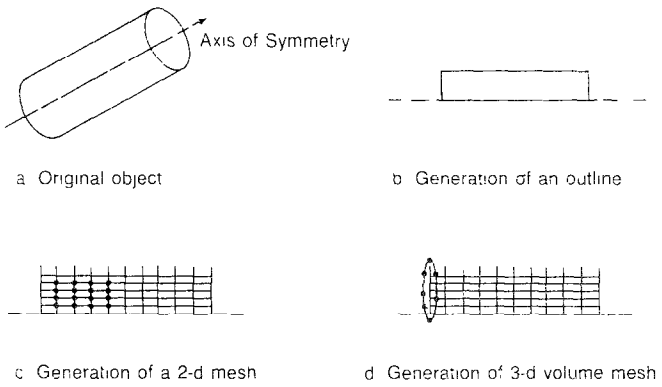


FIG. 2. Generation of a volume mesh.

of the problem space in increments whose length depends on a user-defined resolution factor. At each point along the axis, perpendicular rays are generated. Mesh points are spaced along all segments of these rays that are found to be inside the problem space. When the entire 2D mesh has been generated, each point is rotated about the axis of symmetry, defining new mesh points at appropriate intervals during the rotation, resulting in a 3D mesh. Each of the set of mesh points generated in this way by one of the points in the 2D outline belongs to a single identifiable volume ring whose axis is the same as the overall axis of symmetry mentioned above. This circumstance will be used later in taking advantage of the symmetry of the problem.

The volume assigned to a volume ring is simply a function of its width and the radius of the inner and outer boundaries of the ring. Each volume element in the ring is assigned an equal volume appropriate to the total number of elements in the ring.

Treatment of Volumetric Emission and Absorption

One must next address the problem of handling emission from and absorption in the various volume elements. In order to simplify this task, it is assumed that the problem space is completely enclosed. An understanding of the advantages of using completely enclosed geometries may be gained by considering an enclosure composed of some chosen number of discrete surface elements, as shown in Fig. 3 [6]

The incoming radiation at surface k is equal to the sum of the contributions from all the other surfaces after allowing for absorption in the intervening medium, plus the contribution due to emission from the gas. In the figure the radiation path between k and another surface j within the incident solid angle $d\omega_k$ is shown. If the contributions from all of the other surfaces, including that from surface k if it is concave, are determined by following the paths between the surfaces; the solid angles swept out will encompass all of the gas or plasma that can radiate to surface

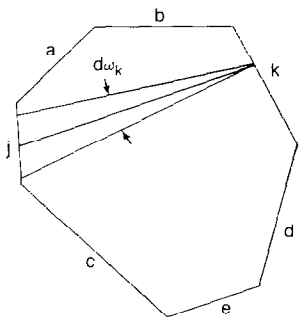


FIG. 3. Gas filled enclosure with discrete surfaces. A typical path from area j to area k is shown.

k. Thus, if the contribution of gas emission at all points along the path between two surface is taken into account for all surfaces, the gas emission for the entire volume will be automatically accounted for. This obviates the necessity for complicated schemes which attempt to handle volumetric emission and absorption as characteristics of individual volume elements. Such schemes normally lead to serious problems due to memory constraints. Instead, the fraction of radiation emission originating in a volume element that should be contributed to each surface element, as well as the amount of radiation in a bundle passing between two surface elements that should be absorbed as it passes through a volume element in its path, can be calculated in advance and assigned to an existing ray between two surface elements.

To see how this is accomplished, let us first consider the question of emission from the medium. The fraction of the total radiation emitted by each volume element in the problem to each surface element must be calculated. It is assumed that the volume elements emit isotropically, although prescription of some angular dependence is not out of the question, just as an angular dependence of some sort might be postulated for the emissive and absorptive characteristics of the surface elements. The contribution from volume element a' in the direction of surface element k in a time step will be

$$E_{a'k} = S_{a'} V_{a'}((A_k \cos\theta)/r_{a'k^2})\, \Delta t, \tag{7}$$

where $E_{a'k}$ is the radiant energy contribution in ergs from volume element a' to surface element k, $S_{a'}$ is the source strength of volume element a' in ergs/cm^3-s, $V_{a'}$ is the volume of element a', A_k is the area of surface element k, θ is the angle between the normal to the surface element and the line between the centroids of the volume and surface elements, and $r_{a'k}$ is the distance between the elements. Once the contribution of a volume element to a surface element has been found, the contribution is assigned to one of the many bundles of radiation passing between the given surface element and all the other surface elements in the problem. These bundles are the same ones we have described above in the context of vacuum problems. The

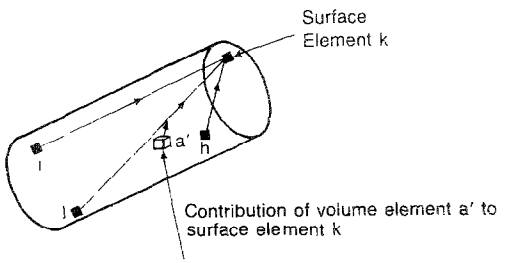


FIG. 4. Representative surface element to surface element rays. The radiative contribution from volume element a' to surface element k is assigned to a bundle on the nearest ray between elements j and k.

paths the bundles take between surface elements, which I will refer to as rays, are generated as a by-product of view factor calculations which have been done earlier by the code. All radiation in the problem, including that generated by the medium, is assumed to travel along one of them. The appropriate volume and surface elements and some of the rays are shown in Fig. 4.

At this point it may be pointed out that there is no need to store in memory the contribution of each volume element to each surface element. The 2D symmetry of the problem space makes it possible to treat the contribution of volume elements as a portion of the contribution of the volume ring to which they belong. Figure 5 illustrates this point. Obviously, the contribution of radiation from volume ring A' to any element on surface ring K will be identical as long as the elements are of

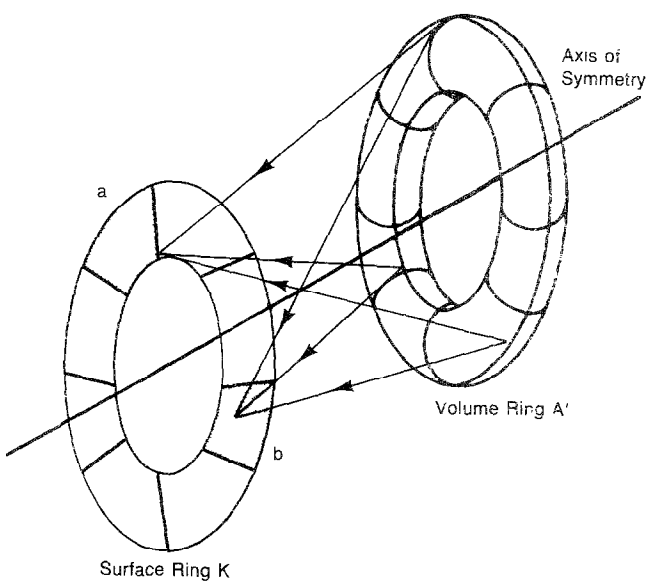


FIG. 5. Contribution of a volume ring is equal to each of the elements of a surface ring.

constant size. Furthermore, rotating the volume ring about the axis of symmetry will not change this contribution, even though individual volume elements are moved about because, thanks to the 2D symmetry of the problem, each volume element in a ring radiates identically.

Further memory savings are realized by taking advantage of the symmetry of the surface rings. Since the radiation received by each surface element in a ring from all sources will be identical for every element in the ring, we need only consider one element per ring as a receiving element. In short, one artificially converts the intrinsically 3D view factor method to 2D for the sake of conserving memory.

Now let us once again consider the bundles of radiation passing along the rays between surface elements. Each of these bundles can be identified unambiguously by specifying the sending element, receiving ring, and the number of time steps that light must travel to reach the location of the bundle. So far, however, we have not considered how to let the code know which volume rings will contribute energy to a bundle in a time step and how much they will contribute. At first glance, this seems an almost insurmountable problem in terms of storage requirements. Apparently, we must create an array large enough to contain an identifier for the volume ring, sending element, receiveing ring, and time step. For a simple problem, assuming reasonable resolution, one might have 200 volume rings, 250 sending elements, 50 receiving rings, and 50 time steps between the two most distant surface elements. Then one would need two or three arrays with $(200 \times 250 \times 50 \times 50)$ or 125 million words reserved for each. Such memory demands may not seem unreasonable in a few years, but at present they are beyond the capabilities of most machines.

Suppose we could assume, however, that the volume elements are sufficiently large and the time steps sufficiently short that a bundle can only pass through portions of three or four volume elements in a time step. One could compute in advance which volume elements contributed to which bundles and use this information to reduce memory requirements. This strategy is used in RAYNA II.

To illustrate the procedure, let us consider the energy emitted from volume element a' in the direction of surface element k. From Eq. (7) we know the actual quantity of energy involved for a given volumetric rate of emission. Since this rate may vary with time, a more appropriate quantity to store is the fraction of the total energy emitted from a' in a time step which is initially directed toward element k. We know the equations of the rays from all the surface elements to element k and the location of volume element a'. The energy contribution from a' is assigned to the closest ray to the volume element. The actual point on the ray where the energy from a' should start contributing is, of course, the nearest point on the ray to the volume element. The approximate location of this point is stored in memory as some number of time steps from the sending surface element, the distance light travels in a time step being assumed constant. As mentioned earlier, we assume that a bundle on any ray can only pass through portions of three or four (or some larger number, memory allowing) volume elements in a time step. Space in memory is reserved for these three or four contributions. If the contribution from a' is the first

to the bundle for the given time step, it is assigned to the first of these spaces, and so on. If all the three or four spaces on the ray are taken, RAYNA II first checks to see if memory is still available. If so, it generates a new ray through the volume element a' to surface element k. A "virtual sending element," j, is declared at the point along the ray from k in the direction of a' at which it intersects some other point on the surface. This virtual surface element cannot emit radiation, and serves only as a starting point for the ray. In this manner, it is possible to achieve high resolution at points of interest in the medium where the user might choose to refine the volume mesh. If memory is exhausted, RAYNA II checks a set of 10 next-nearest alternate points to assigne the radiation from a'. If all these are also found to be full, it declares the array space exhausted and stops the run.

Let us next turn to the task of dealing with volumetric absorption. Suppose we have some physical model for determining absorption coefficients in the medium. In addition to finding the identities of the volume elements through which a bundle passes in a given time step, a problem we have already dealt with in solving the emission problem, one must find the path length of each bundle of radiation in these elements. With this information, one can calculate the amount of energy absorbed in each volume element per time step. We know the fixed distance light travels in a time step, Δs, and the number of volume elements through which a bundle passes in a given time step. Some fraction of Δs, then, must be assigned as the path length in each of the elements. These fractional path lengths are determined as follows.

Radiation can travel from the emitting to the receiving element along any ray which intersects both elements at any point on their surfaces. Taken together, these rays generate a volume or "sheaf" inside which all rays traveling between the elements must remain. Assuming constant emission over the surface of the emitting element and that the surface elements are sufficiently small, radiation intensity from the sending to the receiving element within this sheaf is constant. The intersection of the sheaf volume with that of a volume element, then, should be proportional to the time radiation, between the two surface elements, spends in the volume element and, therefore, to the desired fractional path length. To facilitate the calculations, the sheaf volume is approximated by that of a cone with cross sections at the sending and receiving elements equal in area to the projections of the surface element areas on the plane normal to the line connecting the surface elements. The intersection of the volume of this cone with that of the volume elements encountered in a given time step is determined, and the sum of these quantities is normalized to one, giving the fraction of the total path length spent in each of the volume elements.

Energy absorbed in a volume element, a', from a bundle, B, may then be given as

$$E_{\text{abs}}(a') = E_{\text{B}}\{1 - \exp(-\alpha_a F_p(a') \Delta s)\}, \tag{8}$$

where E_B is the total energy in the bundle, $\alpha_{a'}$ is the absorption coefficient in a' in units of cm^{-1}, and $F_p(a')$ is the fractional path length in element a' as calculated above.

At the moment, scattering effects are not included in the RAYNA II code. However, isotropic scattering could be modeled easily as absorption followed by instantaneous isotropic re-emission. Non-isotropic scattering would be more difficult, but certainly not impossible to handle.

To get a better picture of how the algorithm works, suppose the surfaces defining the problem have been broken down into surface elements and the intervening medium has been divided into volume elements. Two arbitrary surface elements, i and j, and the ray between them, are shown in Fig. 6.

The ray is divided into segments, with each segment representing the distance traveled by radiation emitted by element i as it passes along the ray to element j during each succeeding time step. The length of each segment except the last is, of course, Δs, the distance light travels in a time step. At the beginning of each new time step, one starts at the beginning of the segment, which we shall call segment n, nearest to the receiving surface element, j. The amount of energy in the bundle which arrived in segment n from segment $n-1$ during the last time step is known, as is the total emission in each of the rings, and the absorption coefficients therein. Also known are various quantities which are a function of the geometry of a given problem, such as the identities of the volume rings through which the bundle must pass before it arrives at surface j, the fraction of the total emission from each of those rings which the bundle is to receive during the time step, and the path length of the ray through the rings. These "previously determined" quantities can be computed once and for all at the start of a run or even stored in tables for geometries run repeatedly and will only change if the problem boundaries are altered. With the aid of the above information, we can determine what happens to the bundle at the start of segment n, which we will identify as bundle B, as it passes along the ray to receiving surface j. Suppose volume ring A' is the first one encountered by the bundle. The identity of the ring and the fraction, $f_E(A', B)$, of the known total energy, $E_{\text{tot}}(A')$, which it emits during the present time step which should be contributed to the bundle, have already been determined as described above. The energy contributed by the ring to the bundle, then will be

$$E_{\text{in}}(A', B) = E_{\text{tot}}(A') * f_E(A', B). \tag{9}$$

The energy $E_{\text{abs}}(A'. B)$ which should be absorbed in ring A' from the bundle can

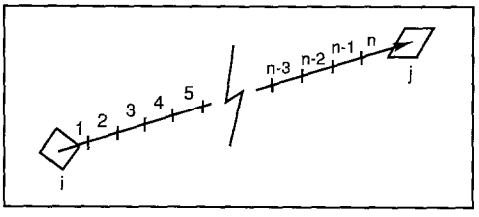


FIG. 6. Typical ray between surface elements.

be derived from the known number of elements in the ring and Eq. (8) above. As bundle B passes through ring A', then, one sets the energy in the bundle E_B to

$$E_B = E_B + E_{in}(A', B) - E_{abs}(A', B). \tag{10}$$

A similar procedure is followed for each of the other volume rings that the bundle encounters on its path.

The procedure is altered somewhat to deal with optically thick regions. At the beginning of each volume ring encountered, the path through the ring is broken down into segments of not greater than one tenth of a mean free path in length. The calculation is then carried out as outlined above with the exception that absorption from and emission into the bundle are updated at the end of each of these segments instead of at the point where the bundle leaves the ring. In this way, errors due to excessive self-absorption in a ring are avoided.

Having dealt with the bundle passing through segment n, one next turns to the bundle which must traverse segment $n-1$ and update its energy content in a similar fashion as it passes along the ray to segment n. This becomes the new bundle energy at the start of segment n for the next time step. One iterates backwards along the ray in this fashion until one reaches surface element i. The energy of the bundle as it leaves this element is computed based on the known surface emissivities, reflectivities, and the amount of radiation received from all the other elements during the last time step, as well as the view factor from surface element i to element j, and it is updated as it passes along the ray through segment 1 just as was done for the other bundles. This procedure is then followed for all the other pairs of surface elements in the problem which are visible from each other. After updating volumetric and surface radiative characteristics, one is then ready to proceed to the next time step.

One can get some idea of the memory requirements of the method by considering the problem of labeling each of the volume rings which is to contribute to a given bundle during a time step. First, of course, one must know which time step. Information must be retained for the number of time steps it takes for a bundle to pass along a ray between the two surface elements which are most distant from each other. We will assume 30 time steps is sufficient to resolve the time dependence of the problem. Next, one must identify the sending surface element and the receiving surface ring. Suppose the problem has been broken down into 300 surface elements and 30 surface rings. Finally, one must know in what order the bundle is to encounter the volume rings as it passes along the ray. Let us say a maximum of four such rings may be encountered by any bundle in a time step. We must then reserve $30 \times 300 \times 30 \times 4$, or over one million words of memory to store the array. Two more arrays of similar size are needed to store the path lengths of the bundle in each of the volume ring it encounters and the fraction of the total emission of the ring which should be contributed to the bundle. These three arrays are the largest needed by a considerable margin, and it has been found that problems of reasonable complexity can be solved with a total available memory of around four

million words. The sample problems described later in this paper required around two million words.

Although the geometry used in RAYNA II is somewhat involved, the concept is simple. One simply follows bundles or packets of radiation through materials and between surfaces of known radiative characteristics. The physical transparency of the code makes it possible to apply complex boundary conditions easily.

4. Sample Problems

Assuming gray conditions, the radiative flux from a hemispherical body of gas to an area at the center of its base is [7]

$$q = [1 - \exp(-\alpha R)]\, e_b, \tag{11}$$

where α is the absorption coefficient of the gas in cm^{-1}, R is the radius of the hemisphere, and e_b is the blackbody emissive power in ergs/cm^2-s. The simple form of Eq. (11) for hemispheres has led to the definition of so-called "mean beam lengths" for other geometries. These are approximate values of R which give correct values of q for a particular geometry. We will agree with the notation of Ref. [7] in letting L_e represent mean beam length. Approximations of L_e for numerous geometries are available in the literature. For a geometry with mean beam length L_e, then,

$$q = [1 - \exp(-\alpha L_e)]\, e_b. \tag{12}$$

Since one more often deals with volumes of gas which are in thermal equilibrium with some given absorption coefficient than with black media, we will use

$$d^3Q_e = 4\alpha e_b\, dV\, d\lambda, \tag{13}$$

TABLE I

Values of Mean Wall Flux for a Hemisphere to the Center of Its Base

Absolute coefficient α (cm^{-1})	Mean flux	
	Analytic	Code
1.0*E*–5	2.50*E*1	2.64*E*1
1.0*E*–3	2.38*E*1	2.48*E*1
5.0*E*–3	1.97*E*1	2.02*E*1
1.0*E*–2	1.58*E*1	1.57*E*1
5.0*E*–2	4.97*E*0	5.02*E*0
1.0*E*–1	2.50*E*0	2.52*E*0

Note. Emissive power of medium equals 1 erg/cm^2-s. Flux in units of erg/cm^2-s.

where d^3Q_e is the energy emitted by a volume element in equilibrium with its surroundings. For a small volume in a gray medium of known absorption coefficient and emissive characteristics, we can approximate e_b, then, as

$$e_b = Q/4\alpha, \tag{14}$$

where Q is volumetric emissive power in ergs/cm^3-s.

In what follows, the fixed value $Q = 1$ is legislated and the absorption coefficient is varied. The approximate value of e_b from Eq. (14) is used in Eq. (12) to find q. Values of q found in this way are compared with RAYNA II results for two different geometries in Tables I and II, namely, the cases of a hemispherical body of gas radiating to the center of its base and a circular cylinder of height equal to its diameter radiating to an element at the center of its base. Both geometries are shown in Fig. 7.

The results for a hemisphere of gas radiating to a point at the center of its base shown in Table I were obtained for a hemisphere of radius 100 cm. The base of the hemisphere was composed of a ring of outer radius 100 cm and inner radius 1 cm, and a disk of radius 1 cm to serve as the "differential" area at the center of the base. The hemisphere and ring were divided by the code into 50 and 60 elements, respectively, and the volume was broken down into 52 rings with a total of 1073 volume elements. There were 30 time steps between the two most distant surface elements in the problem. All problems were run in single precision, with the exception of a few variables for which better accuracy proved indispensable.

In Table II are shown the results for a cylinder of height equal to its diameter, D, radiating to an element at the center of its base. In this case, four surface regions were used: a cylinder, a large disk at the top, a large ring at the bottom, and a small disk at the center of the bottom ring from which the results were obtained. The cylinder, large disk, and large ring were divided into 60 surface elements and the small disk was divided into four. The problem space was broken down into 66

TABLE II

Values of Mean Wall Flux for a Circular Cylinder of Height Equal to Diameter, D, Radiating to an Element at the Center of Its Base

Absolute oefficient B α (cm^{-1})	Mean flux	
	Analytic	Code
1.0E–5	1.92E1 ($L_e = 0.77D$)	1.95E1
1.0E–3	1.85E1 ($L_e = 0.77D$)	1.85E1
5.0E–3	1.55E1 ($L_e = 0.74D$)	1.55E1
1.0E–2	1.27E1 ($L_e = 0.71D$)	1.29E1
5.0E–2	4.86E0 ($L_e = 0.71D$)	4.64E0
1.0E–2	2.50E0 ($L_e = 0.71D$)	2.39E0

Note. $Q = 1$ erg cm^3-s; flux in units of ergs/cm^2-s.

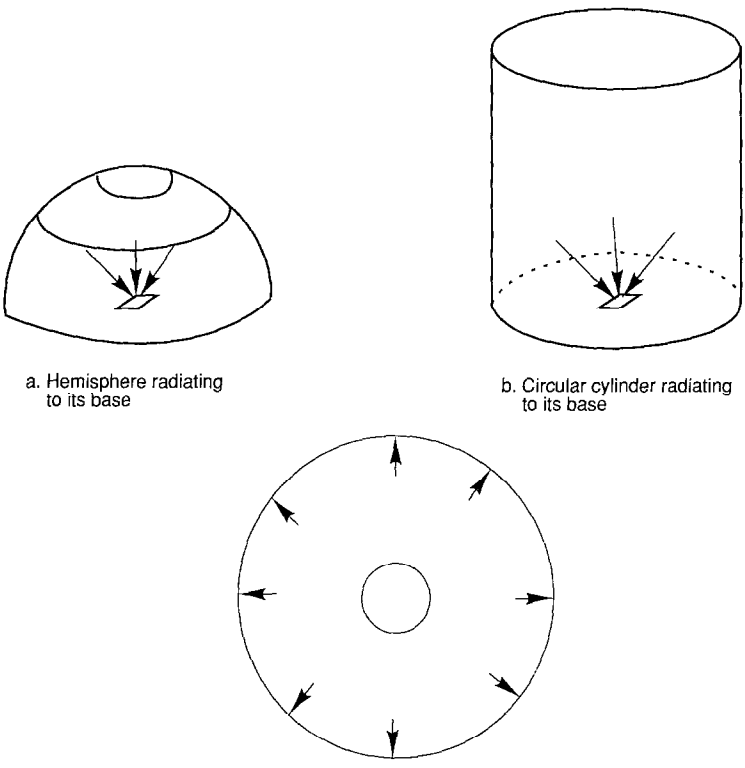


FIG. 7. Geometries for sample problems.

volume rings with a total of 1155 volume elements. There were 35 time steps between the two most distant surface elements in the problem. Mean beam length for this geometry is given as 0.77D for optically thin gases, decreasing to 0.71D for media of significant optical thickness. Mean beam lengths used in the code are shown on the table.

The problem of concentric spheres, also shown in Fig. 6, is presented as a demonstration of the method's ability to handle time-dependent solutions. In particular, let us consider the case in which the surface of the central sphere begins radiating at some constant strength at time $t = 0$. One wishes to solve for the time-dependent, radially outward directed flux at the surface of the outer sphere. Analytic solutions to the vacuum problem are available in the literature [2] for the case in which the central sphere has become so small it may be approximated as a point source. To my knowledge, non exist for the case of a participating medium with arbitrary absorption coefficient other than those presented in Table III and Fig. 8 below. These results are for inner and outer sphere radii of 0.1 cm and 100 cm, respectively.

The results are expressed in terms of t_c, the light travel time across the outer sphere, and S_0, the effective radially outwardly directed source strength at the surface of the outer sphere. Surface albedoes (reflectivities) were set at 0.5 in all cases. In order to improve resolution, the step function source was initiated at the surface of the outer sphere at time $t=0$, rather than at the inner sphere, as in [2]. For the case of an albedo of 0.5, then, the (inward directed) source strength at the outer surface was set to $S_0/2$, since only half of the incident radiation is reflected. The quantity solved for is the outward directed flux at the outer boundary, $F(t)$. Both spheres were broken down into 125 surface elements, and the region between them was broken down into 49 volume rings with a total of 660 volume elements. Results are given for a considerable spread in absorption coefficients, demonstrating the versatility of the method.

As can be seen from the tables, the code and analytic results were in good agreement for all the problems described above, given the relatively crude subdivision of problem surfaces and volumes. Little deterioration is evident in the performance of the code for problems ranging from optically thick ($\alpha = 1.E-1\ \text{cm}^{-1}$) to optically thin ($\alpha = 1.E-5\ \text{cm}^{-1}$). Unfortunately, because of the recent development of this method, quantitative techniques for estimating error are still unavailable. However,

TABLE III

Radially Outward directed flux $F(t)$ at the Surface of the Outermost of Two Concentric Spheres

i. $\alpha = 1\times10^{-15}\ \text{cm}^{-1}$			ii. $\alpha = 1.0\times10^{-4}$	
	$F(t)/S_0$		$F(t)\cdot S_0$	
t/t_c	Analytic	Code	t/t_c	Code
0.1665	1.014	1.014	1.111	1.014
0.2667	1.036	1.031	1.778	1.030
0.3333	1.056	1.052	2.222	1.052
0.4000	1.081	1.078	2.667	1.077
0.5000	1.128	1.128	3.333	1.127
0.6000	1.186	1.181	4.000	1.179
0.6667	1.231	1.225	4.444	1.223
0.7333	1.281	1.277	4.889	1.275
0.8333	1.368	1.360	5.556	1.356
0.9000	1.433	1.427	6.000	1.421
1.0000	1.543	1.532	6.667	1.525
1.0667	1.556	1.543	7.111	1.536
1.1667	1.579	1.565	7.778	1.558
1.2333	1.597	1.582	8.222	1.574

Note. Space between the spheres is filled with medium of absorption coefficient α. Step function source directed outward with strength S_0 erg/cm^2-s at the surface of the outer sphere turns on at time $t=0$. Times in terms of t_c, the light travel time across the outer sphere

TABLE III—*Continued*

iii. $\alpha = 1.0 \times 10^{-3}$		iv. $\alpha = 2.5 \times 10^{-3}$		v. $\alpha = 5.0 \times 10^{-3}$	
t/t_c	$F(t)/S_0$ Code	t/t_c	$F(t)/S_0$ Code	t/t_c	$F(t)/S_0$ Code
1.111	1.014	1.111	1.013	1.111	1.012
1.778	1.029	1.778	1.028	1.778	1.025
2.222	1.050	2.222	1.047	2.222	1.041
2.667	1.074	2.667	1.068	2.667	1.059
3.333	1.119	3.333	1.108	3.333	1.091
4.000	1.167	4.000	1.148	4.000	1.121
4.444	1.205	4.444	1.180	4.444	1.144
4.889	1.251	4.889	1.217	4.889	1.170
5.556	1.322	5.556	1.273	5.556	1.208
6.000	1.378	6.000	1.316	6.000	1.236
6.667	1.465	6.667	1.381	6.667	1.277
7.111	1.474	7.111	1.388	7.111	1.281
7.778	1.492	7.778	1.401	7.778	1.288
8.222	1.505	8.222	1.410	8.222	1.294
vi. $\alpha = 7.5 \times 10^{-3}$		vii. $\alpha = 1.0 \times 10^{-2}$		viii. $\alpha = 2.5 \times 10^{-2}$	
t/t_c	$F(t)/S_0$ Code	t/t_c	$F(t)/S_0$ Code	t/t_c	$F(t)/S_0$ Code
1.111	1.011	1.111	1.011	1.111	1.007
1.778	1.023	1.778	1.021	1.778	1.012
2.222	1.037	2.222	1.033	2.222	1.017
2.667	1.052	2.667	1.045	2.667	1.021
3.333	1.077	3.333	1.065	3.333	1.026
4.000	1.100	4.000	1.083	4.000	1.029
4.444	1.117	4.444	1.095	4.444	1.031
4.889	1.135	4.889	1.108	4.889	1.033
5.556	1.161	5.556	1.125	5.556	1.034
6.000	1.179	6.000	1.137	6.000	1.035
6.667	1.204	6.667	1.153	6.667	1.036
7.111	1.206	7.111	1.154	7.111	1.036
7.778	1.211	7.778	1.157	7.778	1.036
8.222	1.214	8.222	1.158	8.222	1.036

when one considers the relatively small number of elements into which the surface and volume space of the problem were divided, the limited resolution in time, and the fact that the approximations used to find the surface to surface view factors can be improved upon with little difficulty, it can be concluded that agreement of the code results with analytic solutions was excellent.

The particular sample problems treated here were chosen mainly because similar problems are encountered relatively frequently in the literature, not because the code is limited to such simple geometries. As already noted, the code can solve

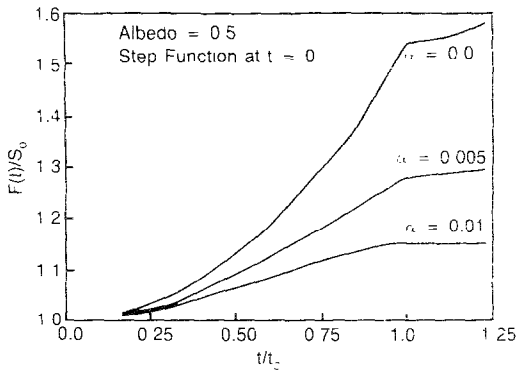


FIG. 8 Outward directed flux at surface of outer sphere for concentric sphere problems.

problems involving arbitrarily varying surface and volume radiative characteristics and source functions.

It is interesting to note that, for the first two geometries given in this paper, the walls did not participate at all except as a radiation sink. This demonstrates the fact that, since the geometric surfaces employed to specify a particular problem need not re-emit radiation, they can play a role similar to that of non-reentrant boundary conditions in diffusion, P_n, S_n, and other radiation transport codes. It is even possible to speculate on the feasibility of "virtual" surfaces, which would neither absorb nor emit radiation and would merely play the role of improving resolution at some point or other in the medium where interesting physics is taking place. Thus, it is quite conceivable that view factor codes may be applied to radiation transport problems involving no real surfaces at all. Furthermore, the code's ability to automatically generate new rays through portions of the volume which the user has artificially refined for better resolution means that the subdivision of the enclosing surface need not dictate the accuracy with which events going on within the

five minutes for convergence to be achieved on an IBM 3031 computer. This did not include the breakdown of the problem into elements, computation of view factors, and other repetitive tasks which can be done once and stored on disk for a given geometry.

5. CONCLUSIONS

The view factor technique described above is a new method of solving non-linear, time-dependent radiative transfer problems for two-dimensional geometries including participating media. The principles applied in the code can be extended without difficulty to three-dimensional problems, assuming the existence of sufficient computer memory resources. The code can be used to solve fully time-dependent

problems in cases where radiation is the dominant mode of energy transfer. It can also be used as an iterative method to provide time-independent solutions to the radiative transfer problem in cases where dominant physical phenomena are occurring on time scales significantly longer than that required for the radiation field to arrive at steady-state or near steady-state conditions. It is felt that the speed and geometric adaptability of codes employing the method will make them very competitive with alternative techniques, such as Monte Carlo, in handling radiative transfer problems with participating media. They are well suited to solution with machines employing parallel architecture, and ongoing improvements in computer memory capacities will enhance their capabilities in the near future, making the treatment of three-dimensional geometries the next logical step.

REFERENCES

1. D. J. DRAKE, KMS Fusion Report No. KMSF-U1668, August 1985 (unpublished).
2. N. A. HOFFMAN, *JSQRT* **34**, 435 (1984).
3. R. C. KIRKPATRICK. University of California LANL Report No. LA-UR-86-1038, 1986 (unpublished).
4. F. KREITH, *Principles of Heat Transfer* (Intext Press, New York, 1973), 243.
5. R. SIEGEL AND J. R. HOWELL, *Thermal Radiation Heat Transfer* (McGraw–Hill, New York, 1972), 170.
6. *Ibid.*, p. 551.
7. *Ibid.*, Chap. 17.